Exploiting surveys of the Milky Way

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Outline

- Our ambition
- Surveys
- The role of dynamical models & simulations
- Equilibrium models, integrability & actions
- Modelling the thin/thick disc interface with GCS & RAVE

Our ambition

- ΛCDM provides the initial conditions from which galaxies formed
- Massive computers provide the means to integrate from these initial conditions
- So we have an elaborate theory of how galaxies like ours formed and predictions for how they are structured
- Are these predictions correct? What *is* the structure of our Galaxy? What does it directly tell us about the structure & evolution of the Universe as a whole?

Surveys

- Near-IR point-source catalogues
  - 2MASS, DENIS, UKIDS, VHS, ….
- Spectroscopic surveys
  - RAVE, SEGUE, HERMES, APOGEE, ESO-Gaia, WHT, …
- Astrometry
  - Hipparcos, UCAC-4, Pan-Starrs, Gaia, Jasmine, …
- Already have photometry of $\sim 10^8$ star, proper motions of $\sim 10^7$ stars, spectra of $\sim 10^6$ stars, trig parallaxes of $\sim 10^5$ stars
- By end of decade will have trig parallaxes for $\sim 10^9$ stars and spectra of 10^8 stars
- We are already data-rich & model-poor

Need for models

- Our position near midplane of disc makes models a prerequisite for interpretation of data
  - Models provide the means to compensate for strong selection effects in survey data
  - Models facilitate compensation for large observational errors
- The complexity of the MW calls for a hierarchy of model of increasing sophistication
  - Axisymmetric model
  - Add the bar
  - add spiral structure
  - Add the warp ….

Relation to cosmological simulations

- We now understand the clustering of DM
- But not the response of baryons because their physics is so complex
- Very small-scale phenomena (accretion discs, magnetic confinement, nucleosynthesis, blast waves) are important for Galaxy-scale structure
- Analogously, the flow of air is determined by how molecules collide with each other
  - At Airbus Industrie they don't simulate dynamics of $>10^{24}$ molecules colliding under QM
  - They use transport coefficients (viscosity, conductivity) measured in the lab
- Cosmological simulations depend on parameters of "sub-grid physics" which are analogous to viscosity etc
- The Galaxy offers an opportunity to "measure" these parameters

On pdfs & realisations

- Models from cosmological simulations are discrete realisations of some underlying probability density function (pdf) – we don’t expect to find a star exactly where the model has one
- The Galaxy is another discrete realisation
- How to ask if 2 realisations are consistent with the same (unknown) pdf?
  - Seems essential to bin one of the realisations
  - Problematic when data high-dimensional (d >= 10) because
  - \# cells = (cells per axis)^d and need many stars/cell
- Much better to formulate the model as a pdf – then can ask if the Galaxy is consistent with this pdf – or in what respects the Galaxy materially differs from it – by calculating likelihoods
- Hence we reject N-body & similar models

Equilibrium models

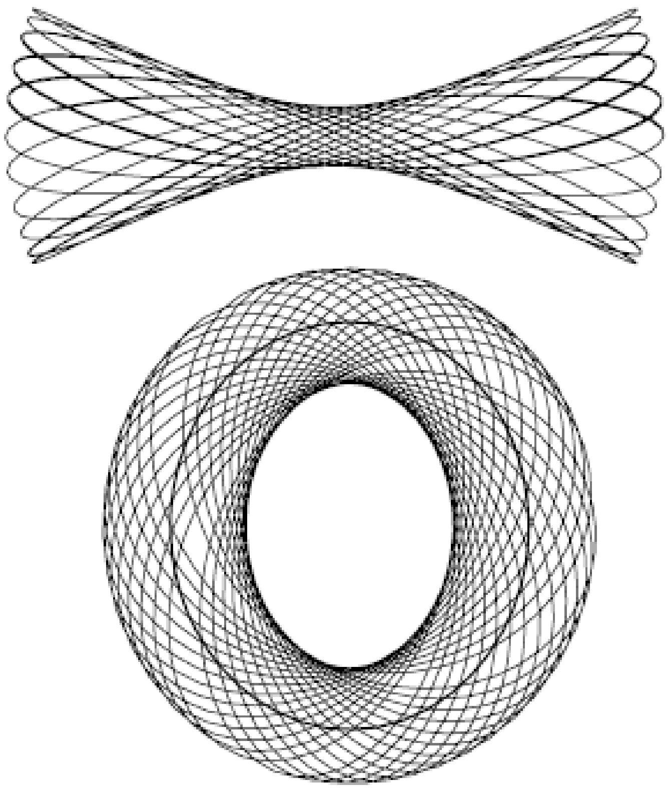
- The galaxy is not in perfect equilibrium
- But we must start from equilibrium models:
  - First target is Φ(x), which will be an important ingredient of our final model
  - Without the assumption of equilibrium, any distribution of stars in (x,v) is consistent with *any* Φ(x)
  - From Φ(x) we can infer ρ_{DM}(x)
  - Can only infer ρ_{DM}(x) to the extent that the Galaxy is in dynamical equilibrium
- Non-equilibrium structure (spiral arms, tidal streams,..) will show up as differences between the best equilibrium model and the Galaxy
- The Galaxy is not axisymmetric, but it is sensible to start with axisymmetric models for related reasons

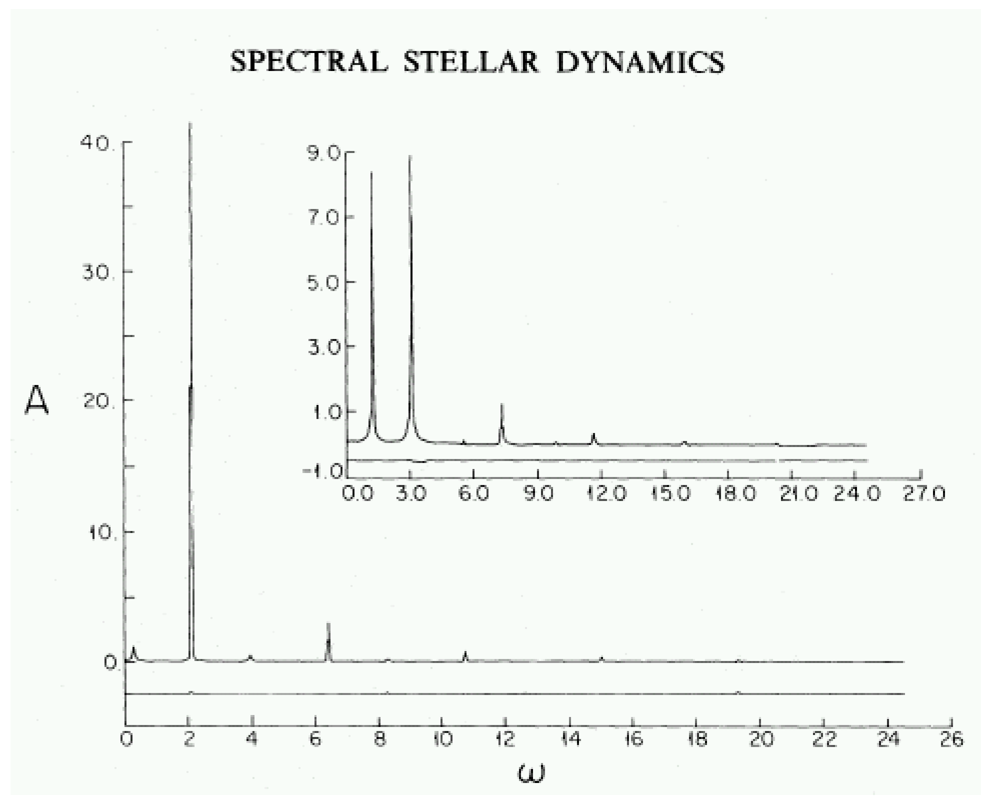
Jeans Theorem

- An equilibrium model can be assumed to have DF f(x,v) = f(isolating integrals)
- Since any function g(isolating integrals) is itself an isolating integral, there is ∞ choice of integrals
- Some integrals stand out: the actions J
- These alone can be embedded in a system (θ,J) of canonical coordinates

Integrability, tori, actions

- Have time-independent $H = \frac{1}{2}p^2 + \Phi(x)$
- Suppose $\Phi \propto \ln(x^2+y^2/q_1^2+z^2/q_2^2)$ & integrate orbits
- Orbits come in families
- Time series x(t) etc are quasiperiodic

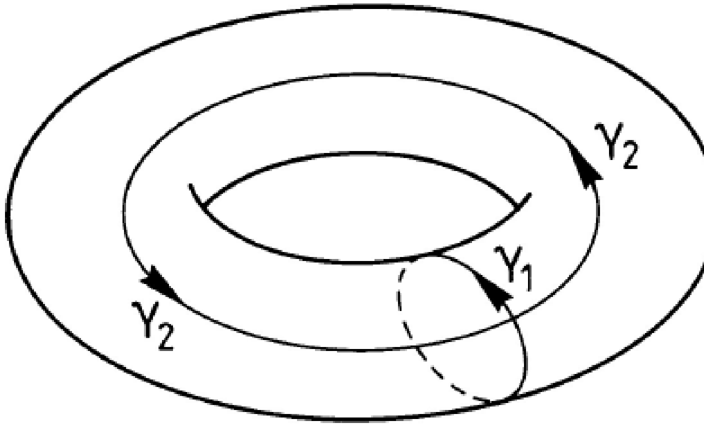




Quasiperiodicity implies n isolating integrals (Arnold)

- Quasiperiodic motion in a Hamiltonian with n coordinates admits n isolating integrals
- Orbits effectively surfaces: Integrals = const
- These surfaces are topologically n-tori in 2n-d phase space

Angles & actions



Binney & Tremaine, Galactic

- Choose n closed paths γ_i around T that cannot be deformed into one another
- Define action $J_i=(2\pi)^{-1}\int_{\gamma_i} dq.p$
- Then conjugate coordinates θ_i exist that give position within torus
- Note on torus T
  - $S = \sum_i\int dp_i dq_i = \sum\int dJ_i d\theta_i = 0$; tori are *null*
- Fact: *any null n-torus in H = const is an orbit*
- Position within T is specified by n angle variables θ_i
  - The θ_i increase linearly in time: $\theta_i(t)=\theta_i(0)+\Omega_i t$
- T is labelled by its action integrals $J_i = (2\pi)^{-1}\int_{\gamma_i} p.dq$, which are specified up front
- In an axisymmetric Φ, L_z is one of the actions

Advantages of actions

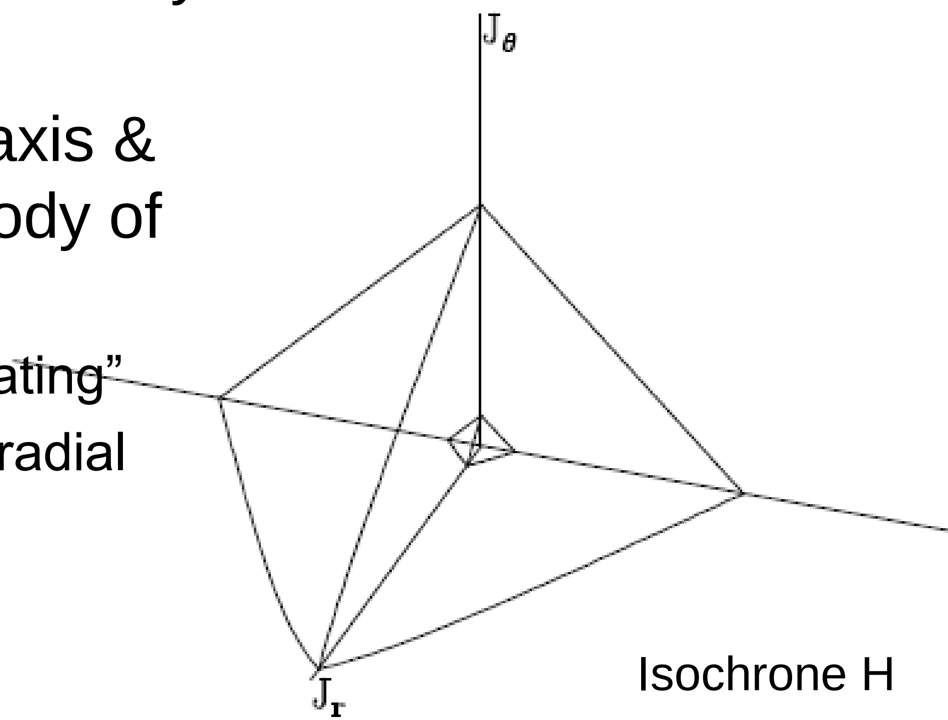
- Action integrals:
  - Are essentially unique
  - Are Adiabatic invariants
  - Have clear physical interpretation
  - Make integral (action) space a true representation of phase space: $d^3x d^3v = (2\pi)^3 d^3J$
  - Make choice of analytic DF easy
- Knowledge of the θ_i of stars key to unravelling mergers (McMillan & Binney 2008)
- Angle-action variables (θ,J) are the key to Hamiltonian perturbation theory
- The only problem: how to compute actions?
- Several schemes are possible. Here we discuss 2 complementary schemes: torus mapping & the Staeckel Fudge (Binney 2012)

Physical meaning of J_i

- Axisymmetric case
  - $J_\phi = L_z$ conserved angular momentum
  - J_z controls amplitude of motion $\perp$ plane; in spherical Φ, $J_z = L - |L_z|$; in epicycle approx $J_z = E_z/\nu$
  - J_r controls amplitude of radial motion; in epicycle approx $J_r = E_r/\kappa$
- Triaxial case
  - Box orbits
    - J_x amplitude of long-axis motion
    - J_y amplitude of mid-axis motion
    - J_z amplitude of short-axis motion
  - Short-axis loops
    - J_ϕ mean angular momentum around z
    - J_R radial oscillations
    - J_z vertical oscillations
  - Long-axis loops….

The DF and action space

- $d^3x d^3v = (2\pi)^3 d^3J$ so $f(J)$ is density of stars in action space
- Surface E=const approximately planar
- Disc stars born near J_ϕ axis & diffuse from there into body of space
  - Diffusion perp to axis "heating"
  - Diffusion parallel to axis "radial migration"



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Binney & Tremaine, Galactic Dynamics

Modelling the thin/thick interface

- Local stellar population can be broken down into
  - A “thick disc” of >10 Gyr old stars with high α/Fe and mostly low Fe/H
  - A “thin disc” with low α/Fe and mostly quite high Fe/H in which SFR has continued for ~ 10 Gyr at a slowly declining rate
- Thick-disc stars have quite large random velocities
- The random velocities of thin-disc stars increase steadily with age

Choice of the DF

- We assemble f(J) of discs from “quasi-isothermal” building blocks

$$f(\mathbf{J}) \equiv \frac{\Omega\nu\Sigma}{2\pi^2 M\sigma_r^2\sigma_z^2\kappa}\bigg|_{R_c} \mathrm{cut}(L_z)\, \mathrm{e}^{-\kappa J_r/\sigma_r^2}\, \mathrm{e}^{-\nu J_z/\sigma_z^2}$$

$$\mathrm{cut}(L_z) = \frac{1}{2}\left[1 + \tanh(L_z/L_0)\right]$$

- Exponential in the actions
- 2 hotness parameters σ_r & σ_z
- $\Omega(L_z)$ the circular frequency
- $\kappa(L_z)$, $\nu(L_z)$ epicycle frequencies
- $L_0 \ll v_c R_0$ unimportant

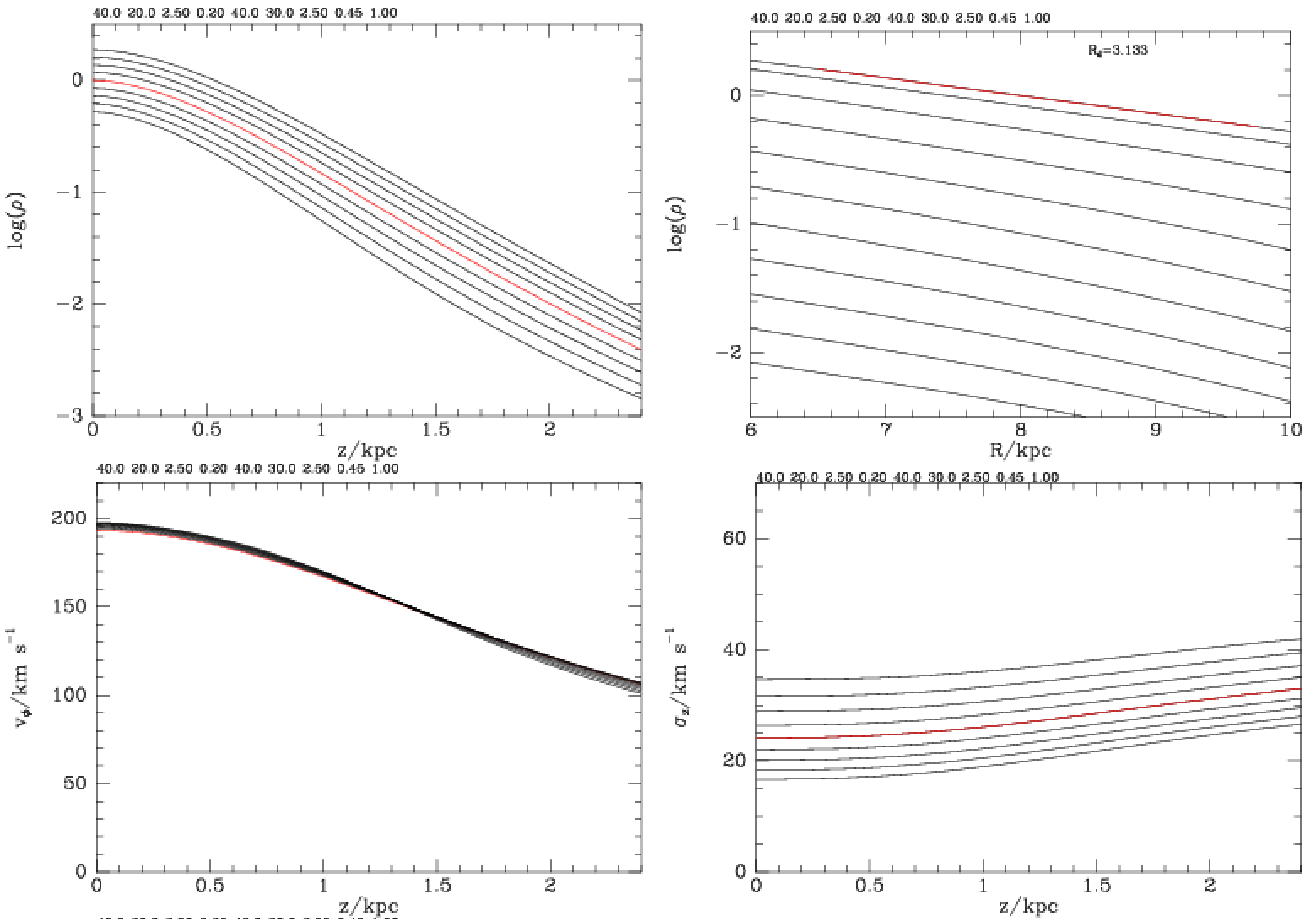
The dispersion parameters should depend on L_z

- Hotness parameters vary exponentially with R

$$\sigma_r(L_z) = \sigma_{r0}\,\mathrm{e}^{(R_0-R_c)/R_\sigma}$$
$$\sigma_z(L_z) = \sigma_{z0}\,\mathrm{e}^{(R_0-R_c)/R_\sigma}$$

- Radial gradient of $\langle v_R^2 \rangle$ etc controlled by R_σ

A quasi-isothermal component



The thin disc is always growing & heating

- Hotness parameters also a function of cohort's age τ

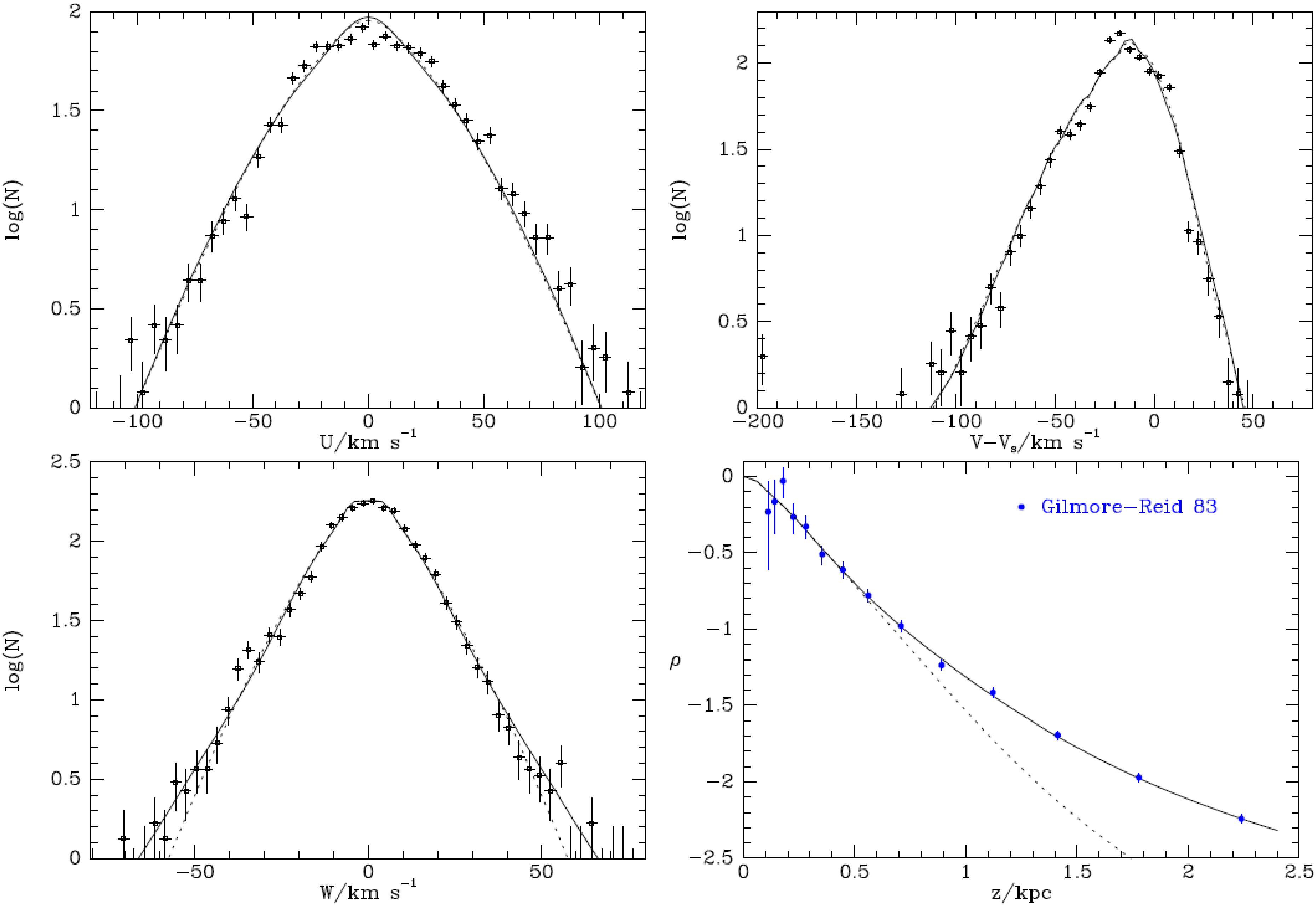
$$\sigma_z(L_z,\tau) = \sigma_{z0}\left(\frac{\tau+\tau_1}{\tau_\mathrm{m}+\tau_1}\right)^\beta \mathrm{e}^{q(R_0-R_\mathrm{c})/R_\mathrm{d}}$$

$$f_\mathrm{thn}(\mathbf{J}) = \int_0^{\tau_m} \mathrm{d}\tau \, \frac{\mathrm{e}^{\tau/t_0} f_{\sigma_r}(J_r,L_z) f_{\sigma_z}(J_z)}{t_0(\mathrm{e}^{\tau_\mathrm{m}/t_0}-1)}$$

- Here we fix β = 0.33, τ_m = 10 Gyr, τ_1 = 10 Myr (Aumer & B 2009)
- Final DF have 9 changeable parameters
  - σ_{r0}, σ_{z0}, R_d, q for thin & thick discs plus fraction of stars in thick disc $f_{thck}/(1+f_{thck})$

Fit DF to GCS v distributions

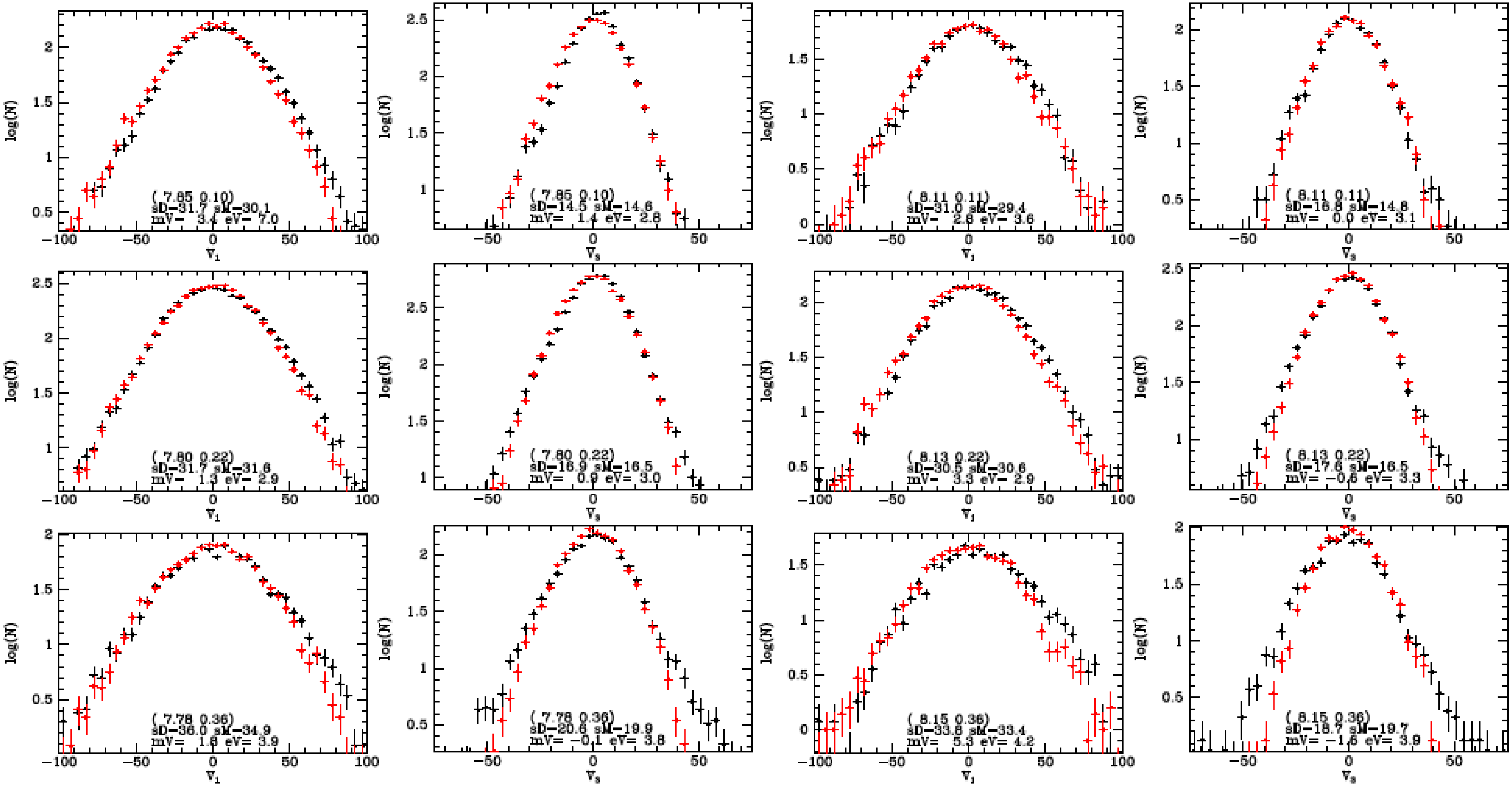
- Use Press et al amoeba to determine the 9 parameters that minimise
  - $\chi^2_{\mathrm{vel}} = \chi^2_{\mathrm{U}} + \chi^2_{\mathrm{V}} + \chi^2_{\mathrm{W}}$
- But GCS UVW don't constrain the thick disc effectively
- So later use Gilmore-Reid $\rho(z)$ & minimise
  - $\chi^2_{\mathrm{rho}} = \frac{1}{2}\chi^2_{\mathrm{vel}} + 3\chi^2_{\rho}$


log(N)
U/km s^{-1}
V−V$_s$/km s^{-1}
W/km s^{-1}
ρ
z/kpc
Gilmore−Reid 83

Predictions for RAVE survey

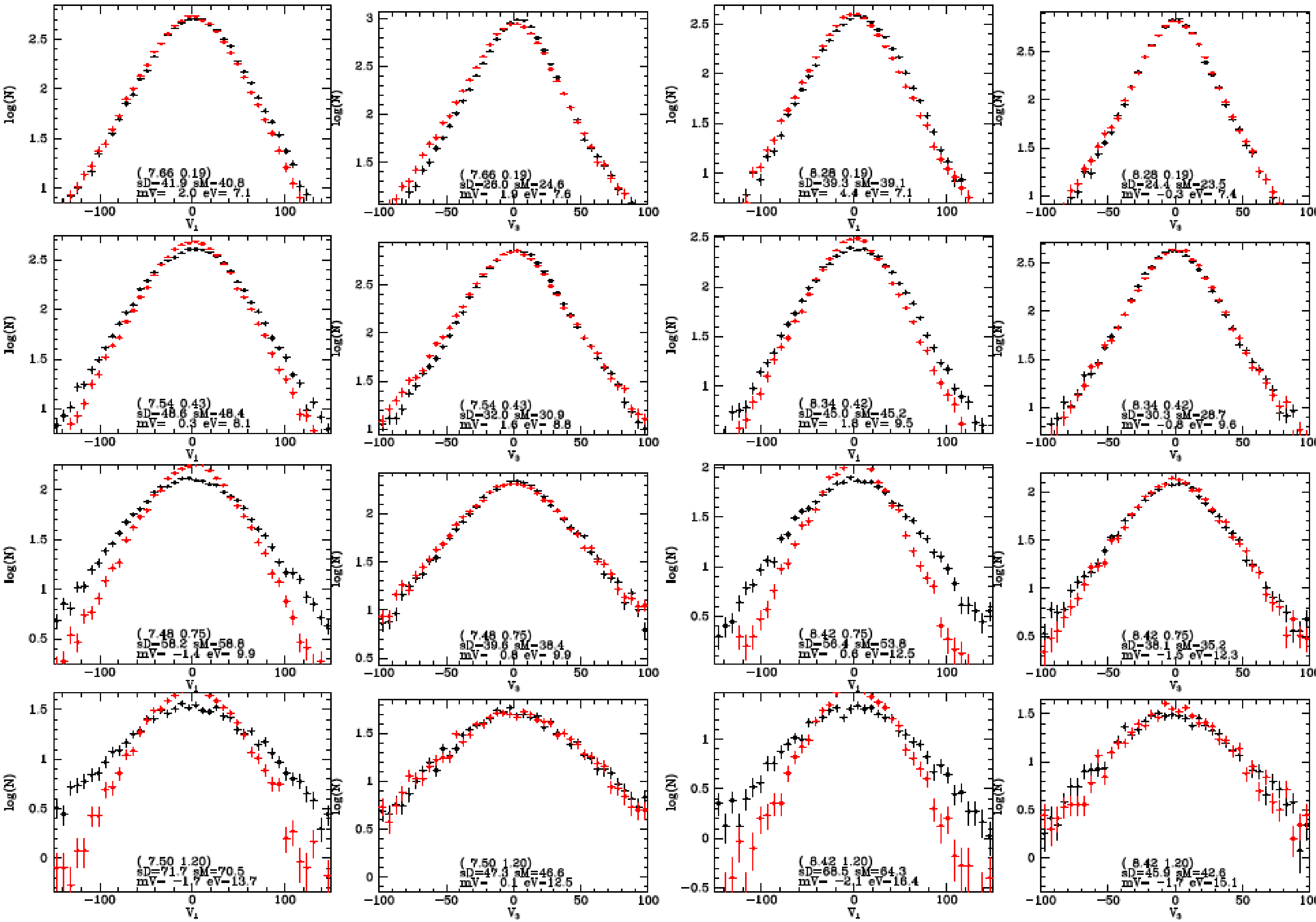
- Rave survey has determined stellar parameters & V_{los} for ~ 400,000 stars
- ~50% giants 50% dwarfs
- Excellent statistics to ~1.5 kpc from Sun
  - GCS extends to ~0.1 kpc from Sun
- For each star in survey choose a new distance (by pdf in distance) & velocity (from model DF), compute proper motion & interpret as v at catalogue distance

Hot dwarfs

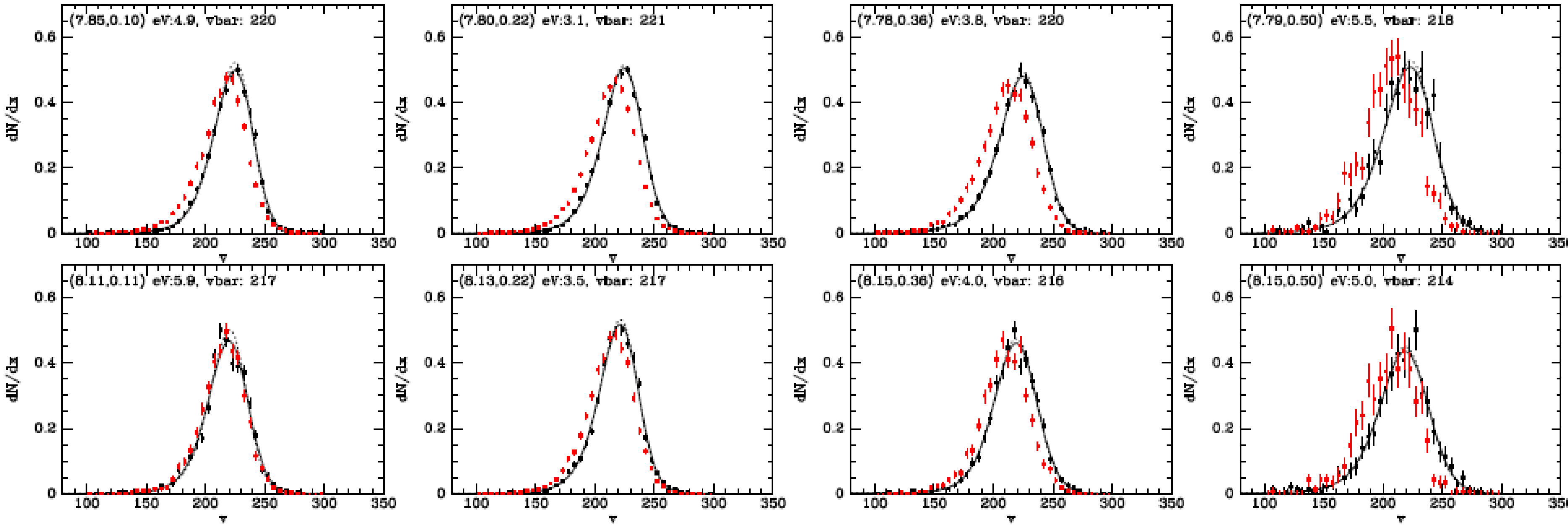


Black: data red: prediction

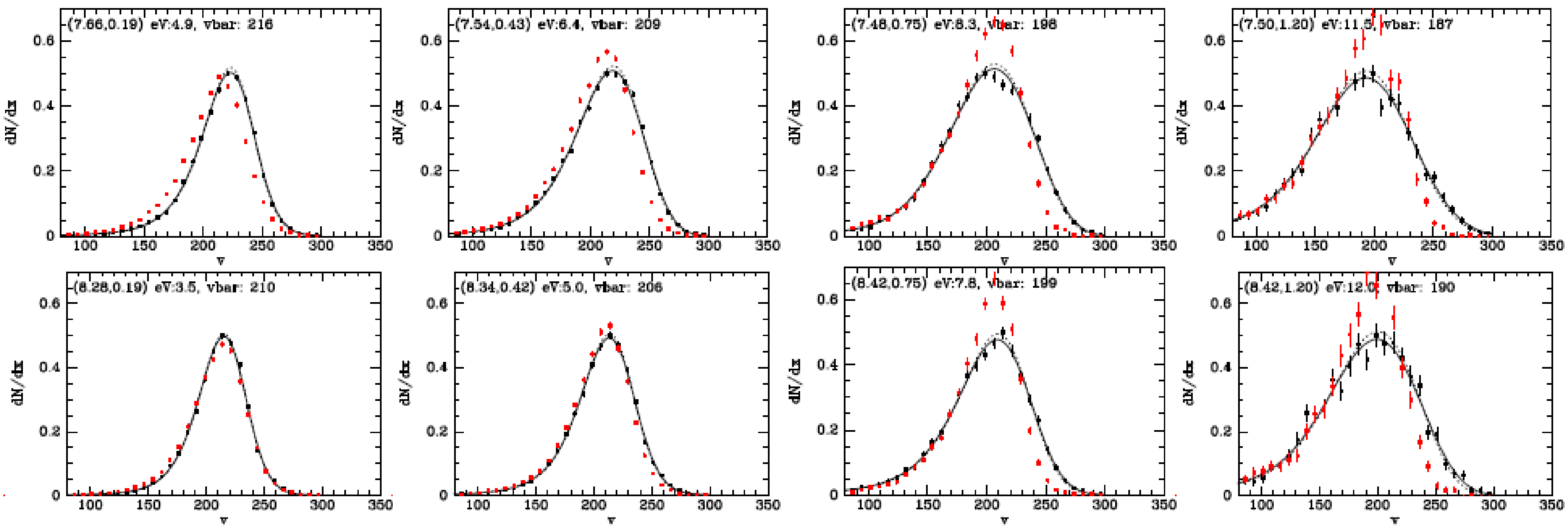
giants


log(N)
V₁
V₃
(7.66 0.19)
sD=41.9 sM=40.8
mV= 2.0 eV= 7.1
(7.66 0.19)
sD=26.0 sM=24.6
mV= 1.9 eV= 7.6
(8.28 0.19)
sD=39.3 sM=39.1
mV= 4.4 eV= 7.1
(8.28 0.19)
sD=24.4 sM=23.5
mV= −0.3 eV= 7.4
(7.54 0.43)
sD=48.6 sM=48.4
mV= 0.3 eV= 8.1
(7.54 0.43)
sD=32.0 sM=30.9
mV= 1.6 eV= 8.8
(8.34 0.42)
sD=45.0 sM=45.2
mV= 1.8 eV= 9.5
(8.34 0.42)
sD=30.3 sM=28.7
mV= −0.8 eV= 9.6
(7.48 0.75)
sD=58.2 sM=58.8
mV= −1.1 eV= 9.9
(7.48 0.75)
sD=39.6 sM=38.4
mV= 0.8 eV= 9.9
(8.42 0.75)
sD=56.4 sM=53.8
mV= 0.6 eV=12.5
(8.42 0.75)
sD=38.1 sM=35.2
mV= −1.5 eV=12.3
(7.50 1.20)
sD=71.7 sM=70.5
mV= −1.7 eV=13.7
(7.50 1.20)
sD=47.3 sM=46.6
mV= 0.1 eV=12.5
(8.42 1.20)
sD=68.5 sM=64.3
mV= −2.1 eV=16.4
(8.42 1.20)
sD=45.9 sM=42.6
mV= −1.7 eV=15.1

Hot dwarfs: v_ϕ



Giants v_ϕ


(7.66,0.19) eV:4.9, vbar: 216
(7.54,0.43) eV:6.4, vbar: 209
(7.48,0.75) eV:8.3, vbar: 198
(7.50,1.20) eV:11.5, vbar: 187
(8.28,0.19) eV:3.5, vbar: 210
(8.34,0.42) eV:5.0, vbar: 206
(8.42,0.75) eV:7.8, vbar: 199
(8.42,1.20) eV:12.0, vbar: 190
dN/dx
v

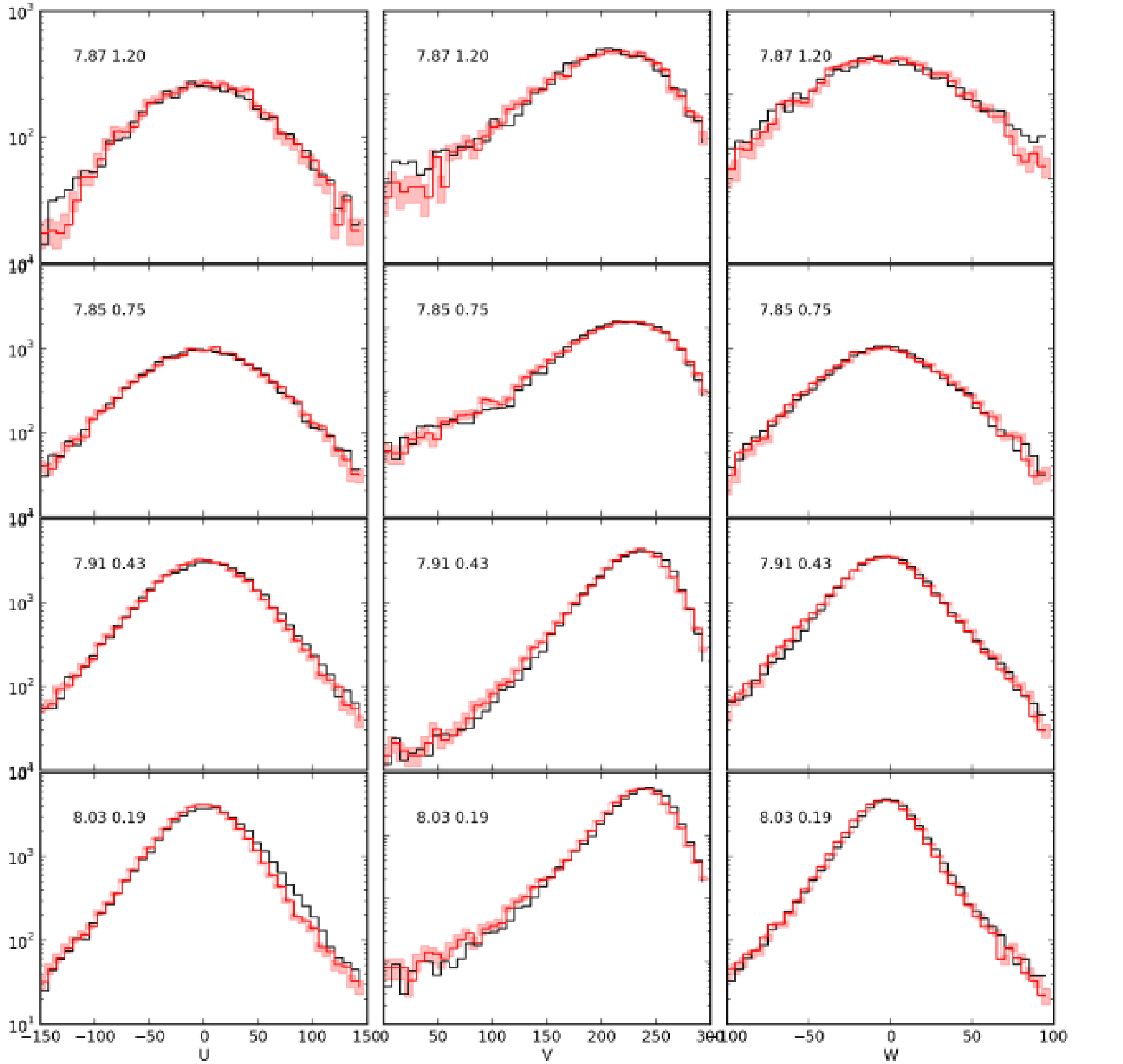
Fitting RAVE

- With pure disc DF can obtain good fits to all kinematics but the disc scale lengths required are implausible
- So we add an isothermal stellar halo $f_{\mathrm{halo}}(E) = \exp\left(\frac{H}{\sigma_{\mathrm{h}}^2} - \eta\right)$

		(a)	(b)	(c)	(d)	(e)	(f)
Thin	σ_{r0}	36.3	37.1	37.2	36.1	35.5	35.4
	σ_{z0}	25.7	26.1	26.4	25.1	26.0	25.2
	R_{d}	4.41	7.31	6.01	3.40	3.81	2.64
	R_{σ}	6.46	8.13	9.80	12.41	10.08	25.17
Thick	σ_{r0}	52.6	53.9	54.5	51.1	49.2	47.4
	σ_{z0}	55.0	55.6	56.9	52.3	53.5	51.2
	R_{d}	18.6	4.72	3.46	2.93	3.69	2.97
	R_{σ}	4.91	8.76	10.15	9.64	8.06	8.53
	F	0.140	0.889	1.219	0.756	0.541	0.397
Halo	σ_{H}	-	95.3	96.4	98.0	94.6	112.8
	η	-	26.113	25.923	26.15	30.55	25.34
	χ^2	6.44	5.73	5.95	7.52	8.25	9.43

Fit to RAVE

- Giants
- Thin
  - (σ_R,σ_z)=(33.7,23.5)
  - R_d=3.22 R_σ= 21.8
- Thick
  - (σ_R,σ_z)=(47.3,50.3)
  - R_d=3.5, R_σ=7.9



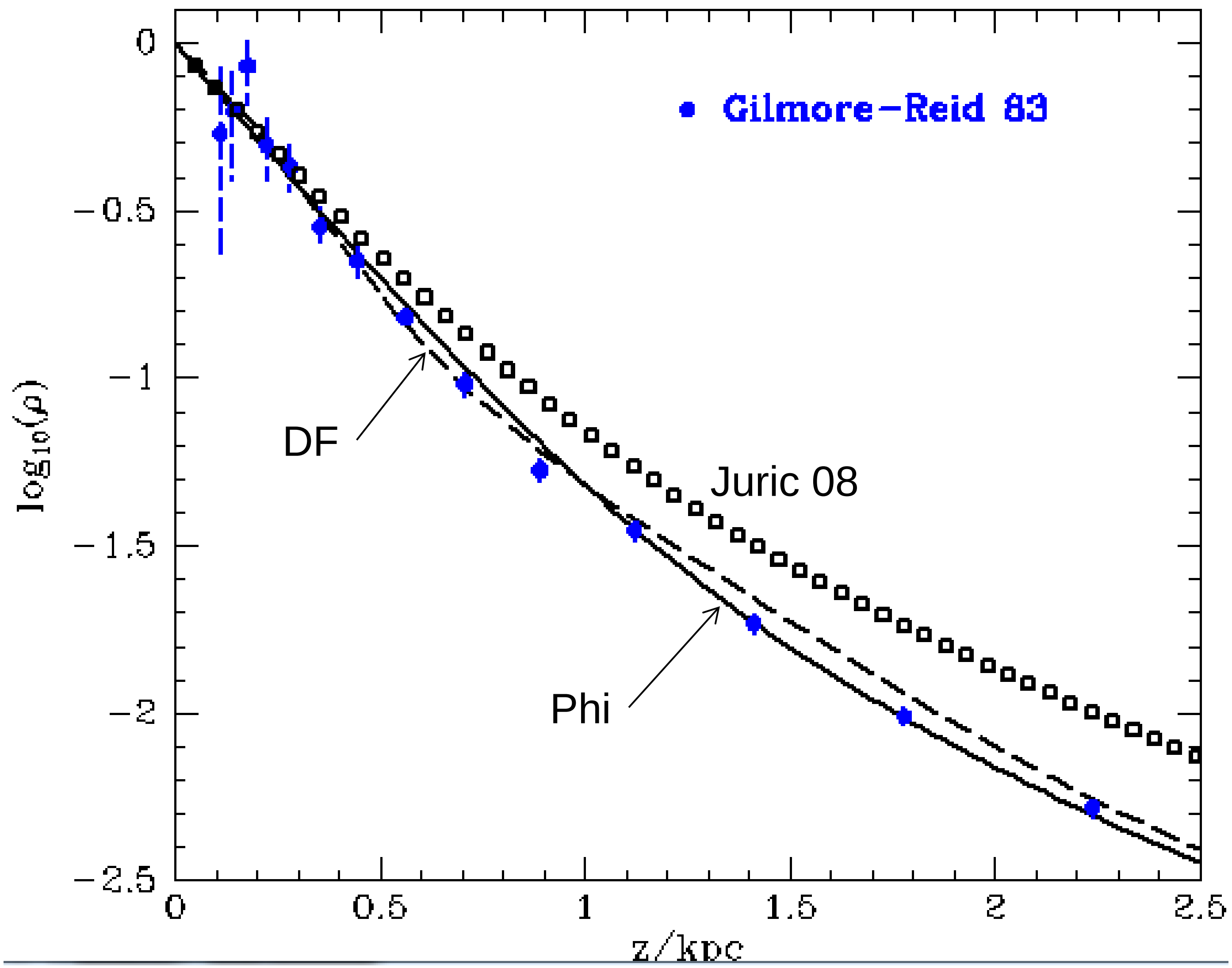
In short..

- For given Φ(x) we can find a DF that provides an excellent fit to RAVE kinematics
- The fitted velocity-dispersion parameters (σ_R, σ_z) of each disc never vary much
- There is strong degeneracy between the scale lengths R_d and R_σ of each disc
- Best to fix R_d at value implied by Φ(x)

Vertical profiles

- Here we are fitting DF to kinematics using a given Φ
- We input stellar & gas discs and a dark halo to derive Φ
- We obtain density of stellar discs as output
- How nearly do in/out agree?

Vertical stellar profiles @ R_0



Next steps

- Currently we are systematically searching for Φ for which the DF that fits the RAVE kinematics yields a vertical stellar profile that's consistent with Φ
- This is a slow process..

Conclusions

- Dynamical models are key for near-field cosmology
- Equilibrium models with f(J) are the most useful
- In the past we lacked ways of computing actions but in the axisymmetric case both J(x,v) and x(J,θ) are available
- Analytic DFs f(J) fitted to GCS kinematics predict RAVE (& SDSS) data with a surprising degree of success
- In any plausible Φ(x) an analytic f(J) can be found that fits RAVE kinematics essentially perfectly
- We expect soon to constrain Φ(x) strongly by requiring that stellar contribution to ρ(z) predicted by f(J) agrees with Φ(x)
- Directions of future work
  - Fit data in their own space (α, δ, π μ_α μ_δ v_{los} J,…)
  - Upgrade DF to f(J,[Fe/H])
  - Add DF of DM and adding self-consistency condition